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Chance and Strategy

Chance and Strategy

Intent

The activities in **Chance and Strategy** are designed to accomplish two goals. First, students are introduced to the game of Pig and have opportunities to begin to develop strategies for playing the game. Second, students encounter several situations that challenge their intuitive notions of probability.

Mathematics

The six possible rolls of a fair die are equally likely to occur. In Pig, a dice game, rolling 2 through 6 will earn a player points. Players may keep rolling until they decide to stop or until they roll 1. Rolling 1 will end the turn, and the score for that turn will be 0. What is the best strategy for playing this game? In other words, at what point should a player stop rolling in order to earn the most points possible for that turn? In **Chance and Strategy**, students begin to test and articulate strategies for optimizing their turns. The idea of a complete strategy—one that can be applied in all possible game situations—is introduced.

Probabilistic reasoning frequently runs counter to intuition. Students often need a great deal of experience before they become willing to base their predictions on probabilistic notions rather than on such ideas as “being lucky” or that a particular result is “due” to occur.

These activities give students some concrete experiences, using dice and coins, with probabilistic questions. The two big ideas underlying these experiences are **independence** (each flip of a coin, for example, is unaffected by previous flips) and **random variability** (it is likely that the result of flipping 50 coins will not be exactly 25 heads).

Students are also introduced to the idea that the probability of an event can be quantified using real numbers between 0 (impossible) and 1 (certain) by counting the occurrences of that event and comparing that to the total number of events possible. The POW introduced in Chance and Strategy, in addition to being another rich problem to solve and to write about, gives students the opportunity to develop systematic counting methods.

There is growing research supporting the idea that technical vocabulary is best learned by students when they attach the terms to concepts they already understand—meaning that it might be advantageous to introduce terminology as students' work on important ideas warrants. To illustrate, consider the following terms in the context of the activity **Waiting for a Double**:

- The **experiment** is rolling a pair of dice.
- The possible **outcomes** of this experiment include rolling 2 and 5, or 1 and 4.
- The **event** of interest is rolling a double (such as 3 and 3, or 6 and 6).
- The **probability** of rolling a double is the ratio of the number of ways this event can occur to the total number of outcomes of this experiment.

Progression

In the opening class and homework activities, students play the game of Pig. They will return to the game later in the unit, once they have developed tools to analyze playing strategies. Subsequent activities in **Chance and Strategy** begin to develop these tools. The first POW of the unit is also assigned.

The Game of Pig

POW: A Sticky Gum Problem

Pig at Home

Pig Strategies

Waiting for a Double

The Gambler’s Fallacy

Expecting the Unexpected

Coincidence or Causation?

What Are the Chances?

Paula’s Pizza

0 to 1, or Never to Always

The Game of Pig

The Game of Pig

Intent

This activity introduces students to the game of Pig and to the study of probability. Students develop, test, and compare strategies for playing the game as they begin to consider the central problem of the unit.

Mathematics

Students will collect data about rolling a die as part of playing the game of Pig. After some experience with the game, they will be able to articulate that a **strategy** is a complete plan of action intended to reach a particular goal.

Progression

Students are introduced to the game of Pig.

Approximate Time

35 minutes

Classroom Organization

Groups of 4, then pairs, followed by whole-class discussion

Materials

Dice

Doing the Activity

After students have read **The Game of Pig** and, in particular, “Your Assignment,” ask a few students to demonstrate how to play the game and how to keep score. Play a few turns as a class to clarify the rules. Be sure students understand the distinction between a **single roll of the die** and a **turn**. Each student should keep a record of the single rolls and the score for each turn. These records will allow students to review how the game progressed and to analyze the outcome of each turn.

After students play several turns in groups of four, have them play a game or two in pairs (as time permits) and experiment with playing strategies.

Discussing and Debriefing the Activity

Ask students to discuss some of the playing strategies they used. A basic strategy defines, for a given turn, when to roll again to try to earn more points and when to stop rolling to not risk losing the points already earned that turn. You might ask each group to post at least one of their strategies. When duplicate strategies appear, request alternate strategies.

After posing the Key Questions, emphasize that in mathematics a strategy is a complete plan of action intended to reach a particular goal. A strategy for playing Pig must tell whether or not to roll again in every circumstance in which there is a choice.

Key Questions

What do you think a strategy is?

How do you use strategies in your daily life?

What were some high scores for individual turns?

How did you decide whether to roll again or to stop?

What do we mean by the “best” strategy?

How did your strategy change as you played Pig and understood the game better?

Does this group’s strategy seem complete and clear?

How can we compare different strategies?

How might you measure whether one strategy is any better than another?

Can another group test your strategy?

POW: The Sticky Gum Problem

POW: A Sticky Gum Problem

Intent

As with all POWs, students explore a problem outside of class and communicate their thinking in writing. This POW taps students' developing skill at finding and using patterns from the previous unit, and asks students to count possible outcomes, a strategy used throughout the current unit to find probabilities. This POW also helps students understand what is meant by a generalization. It also provides a constructive opportunity to understand what distinguishes a POW from an ordinary homework assignment: the focus on generalization and exploration.

Mathematics

At the heart of this activity is the counting concept known as the "pigeonhole principle." Informally, the pigeonhole principle states that if you have more pigeons than pigeonholes, at least one of the holes will contain more than one pigeon. In this activity, the "pigeonholes" are different colors of gum balls. If two children want to buy gum balls of the same color from a machine containing gum balls of three colors, **in the worst case**, the first three gum balls would be three different colors (they will fill the three pigeonholes). The next gum ball would have to match one of the first three. So, the children would have to buy at most four gum balls to be sure to get two of the same color.

Progression

This POW is introduced at the start of the unit, with presentations about a week later.

Approximate Time

10 minutes to pose activity and short periods to check on progress

1 to 3 hours for activity (at home)

20 minutes for presentations

Classroom Organization

Groups of 3 or 4 for brief segments, concluding with whole-class presentations

Doing the Activity

Students can begin working on this POW on the first day of the unit. The activity begins with several special cases and then asks students to search for a general method, given any number of children and any number of colors, for finding the number of gum balls one must buy to be sure the children all get the same color gum ball. A class discussion of the first special case—in which the answer is given and students are asked to explain why it is correct—might help students get started.

Discussing and Debriefing the Activity

After a week or so of having students work outside of class, with perhaps a portion of one day devoted to a progress check, have some students present their results.

Focus the discussion on the process of trying different examples, organizing the information, looking for patterns, expressing patterns symbolically, and explaining patterns, rather than on specific formulas. At this early stage, you might expect only some students to arrive at a general formula.

Key Questions

In each specific case, what is the smallest number of gum balls the children could purchase and all get the same color? Why?

In each specific case, how many gum ball purchases would guarantee that the children would all get the same color? Why?

For any specific number of children, how does the number of colors available affect the number of gum balls one would need to buy? What patterns can help you to answer this question?

For any number of available colors, how does the number of children affect the number of gum balls one would need to buy? What patterns can help you answer this question?

0 to 1 math testing

0 to 1, or Never to Always

Intent

In this activity, students work with probabilities expressed as fractions, percentages, and decimals.

Mathematics

Students continue to focus on the distinction between probability based on a theoretical model and probability based on observed results and on measuring probability using a real-number scale from 0 to 1.

Progression

Earlier in the unit, students estimated the probability of given events. In this activity, they invent situations that match a given probability.

Approximate Time

5 minutes for introduction

25 minutes for activity (at home or in class)

10 minutes for class discussion

Classroom Organization

Individuals or pairs, followed by whole-class discussions

Doing the Activity

Clarify that students are to invent two situations for each question, one based on theoretical probability and the other on observed results. Let's test a link to Keypress.com by clicking here.

1. Here is an equation with a non-breaking space: **x** + **y** = **z**
2. Here is a test of subscripts and superscripts: **x**2 + **y**2 = **z**1 + **z**2

Discussing and Debriefing the Activity

If students have worked individually, ensure that there is time for them to compare their invented situations. Question 5 is included to give students an opportunity to consider an impossible situation.

Key Questions

Does it matter whether probabilities are written as fractions, decimals, or percents?

Why is the smallest probability 0?

Why is the largest probability 1?

What does a probability of 2.3 imply?

Waiting for a Double

Intent

In this activity, students conduct their first simulation, collecting, displaying, and summarizing data to test a conjecture about probability. The activity draws upon their intuitive sense of probability and engages them in thinking more precisely about the topic. This is one of several activities in which intuitive notions about probability can be at odds with experimental or theoretical analyses.

Mathematics

When rolling two dice, what is the average number of rolls needed to roll a double? In this activity, students approach this question through collecting and analyzing data. By rolling two dice repeatedly, they collect many examples of how many rolls it can take to get a double.

Progression

Up to now, students have been exploring the unit problem informally. Now they begin a series of explorations designed to help them develop the tools they will need to answer the unit problem. The in-class portion of this activity has several parts: reviewing the homework, gathering class data, and graphing the class data.

Approximate Time

5 minutes for introduction

20 minutes for activity (at home or in class)

25 minutes to collect and analyze class data

Classroom Organization

Individuals, then small groups to review data, test Equation Editor: $z^2 + 3^3 = \sqrt{5.5}$ + 4. Then the whole class gathers to analyze data. This is a test of Equation Editor: $z^2 + 3^3 = \sqrt{5.5}$ + 4.

$$x^2 + y^2 = z_1 + z_2$$

" $\frac{1}{2}\sqrt{2}^3\mathrm{ACB}\hat{\vec{\mathrm{C}}}$ "

$$\sum_{i=0} n$$

For a discussion of simulating class data, click here. For a Fathom file that can be used as a demonstration of the process followed during this activity, click here.

Key Questions

Before assigning the activity:

How would you do this experiment?

Why is it important to get genuine data when doing activities like this?

How do you compute the average of a set of numbers?

During the class discussion:

How did you make your prediction?

What were the highest and lowest number of rolls you got?

How might you graph one student’s outcomes?

Can you visualize the average in terms of the graph?

How might we graph the outcomes of the entire class?

Is 1 a likely outcome?